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LETTER TO THE EDITOR

# Equivalent isotropic $Osp(2|2)$ spin chains and its Bethe ansatz analysis

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**Abstract.** In this paper we discuss the equivalence between two integrable and isotropic $Osp(2|2)$ chains via the braid-monoid approach. We use the analytical Bethe ansatz approach in order to solve the spectrum of these systems. The ground state is parametrized by a special structure of the Bethe ansatz roots. Their interpretation in terms of correlated fermionic models is briefly discussed.

It is known that the solution of the Yang–Baxter equation plays an important role in the construction of exactly integrable models [1]. Recently, a new rational solution of the Yang–Baxter equation invariant under the $spl(2|1)$ superalgebra has been found in the literature [2–4]. The mean feature of this solution is the presence of an additional non-additive parameter and its explicit invariance under the $U(1)$ symmetry. On the other hand, it is known that a rational $Osp(2|2)$ invariant solution can also be constructed by using many different approaches [5–8]. Due to the isomorphism $spl(2|1) \sim Osp(2|2)$, it is reasonable to think that these two solutions might be connected, for certain particular values of the extra parameter appearing in the $spl(2|1)$ solution. One possible way to verify such a relation is to test whether or not such rational *R-matrices* share common algebraic structures, such as the braid-monoid algebra [9]. The basic problem here is that we checked that the standard $Osp(2|2)$ monoid [8] does not preserve the $U(1)$ invariance. Thus, from the point of view of Boltzmann weights, such a connection is not established in a trivial way. The first purpose of this work is to clarify this issue, by showing that the rational $spl(2|1)$ solution at its fundamental four-dimensional representation [10] can be written in terms of the braid-monoid scheme. We then take advantage of the $U(1)$ and crossing properties of the braid-monoid approach in order to solve the associated $Osp(2|2)$ spin chain by the analytical Bethe ansatz approach. We found that the ground state is determined by a peculiar string structure of the Bethe ansatz roots. We also discuss their connection with models of correlated fermions [4, 11–14].

We start our discussion by recalling the basic properties of the braid-monoid structure appearing in graded rational $R$-matrices (see e.g. [8, 15]). The braid $P_i$ is chosen to be the graded permutation operator $(P_i^g)_{ab}^{cd} = (-1)^{p(a)p(b)}\delta_{ad}\delta_{bc}$, where $p(a)$ is the Grassmann parity. The monoid $E_i$ is a Temperley–Lieb operator [16] satisfying the following extra (braid-monoid) relations [9]†:

$$\begin{aligned} &E_i E_{i\pm1} E_i = E_i \qquad E_i^2 = \xi E_i \\ &P^g_{i\pm1} P^g_i E_{i\pm1} = E_i P^g_{i\pm1} P^g_i = E_i E_{i\pm1} \\ &E_i P^g_{i\pm1} E_i = t E_i \qquad P^g_i E_i = E_i P^g_i = t E_i \end{aligned} \tag{1}$$

where the constant $t$ assumes the values $t = \pm$‡. For the $Osp(2|2)$ system the parameter $\xi$ is zero, $\xi = 0$ [8]. In this case the 'Baxterized ' form [8, 15] of this braid-monoid algebra has the following expression:

$$(R(\lambda)_{1,2})^{cd}_{ab} = \lambda \delta_{ac}\delta_{bd} + (P^g)^{cd}_{ab} - \frac{\lambda}{\lambda - t}(E_{1,2})^{cd}_{ab}. \tag{2}$$

Moreover, in order to search for solutions of the braid-monoid structure (1) we set the following ansatz for the monoid $E_i$ [8]

$$(E_i)^{cd}_{ab} = \alpha_{ab}\alpha^{st}_{cd} \tag{3}$$

where $\alpha_{ab}$ are the elements of a invertible matrix and the symbol $st$ stands for the supertranspose operation. The typical matrix $\alpha$ leading the monoid (3) to satisfy the properties (1) is that appearing on the definition of the $Osp(2|2)$ invariant elements, namely ($bbff$ grading) (see e.g. [8, 7])

$$\alpha_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{4}$$

As it has been mentioned above, we have found out that such a solution breaks the $U(1)$ invariance of the monoid (3) explicitly. However, by imposing the $U(1)$ symmetry, we are then able to find an extra solution which, in the $bbff$ grading, is written as follows:

$$\alpha_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{5}$$

Now we can directly verify that the second solution $\alpha_2$ is equivalent to the $spl(2|1)$ solution of [3, 4] for the special value of the extra non-additive parameter. For instance, in [3] this parameter is denoted by $b$ and one has to take the limit $b \to 0$§. More precisely, considering the appendix of [3], we are able to establish the following identity:

$$(1 + 2u) \lim_{b\to0} R(u, b)^{cd}_{ab} - (I + 2uP^g)^{cd}_{ab} = -\frac{2u}{2u + 1}\tilde{\alpha}_{ab}\tilde{\alpha}^{st}_{cd} \tag{6}$$

where the matrix elements $\tilde{\alpha}_{ab}$ are

$$\tilde{\alpha} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

To complete the verification, one has to set $\lambda = 2u$ and check that the matrix $\alpha_2$ is related with $\tilde{\alpha}$ by just changing the grading $bbff$ to $fbbf$. Next some important comments are in order.

It turns out that solutions $\alpha_1$ and $\alpha_2$ lead us to distinct vertex models‖. In fact, from

† For a generalization of multi-colour versions see, for example, [17].

‡ It is possible to show that the sign of $t$ changes according specific choice of grading.

§ In the case of [4] one has to take (carefully) the limit $\alpha \to -\frac{1}{2}$.

‖ Indeed one can check that the two matrices $\alpha_1$ and $\alpha_2$ are not connected by a unitary transformation.

(2)–(5), we notice that the first model possesses 38 non-null Boltzmann weights while that defined by $\alpha_2$ has only 36 vertices. Besides that, it is evident that they also possess some vertices with different functional behaviour. However, one would expect that common features may appear in a structure involving only the braid-monoid operators. For instance, this is the case of the two-body Hamiltonian defined as the derivative of the $R$-matrix at $\lambda = 0$. In general, the Hamiltonian on a lattice of $L$ sites is given by

$$H_{1,2} = \sum_{i=1}^{L} [P^{g}_{i,i+1} - (E_{i,i+1})_{1,2}] \,. \tag{8}$$

In fact, we have verified by numerical diagonalization of (8) (for several values of $L$, including odd values) that these two Hamiltonians share the same spectrum. This information is extremely useful in order to choose a more symmetrical system ($H_2$) to perform exact calculations such as the Bethe ansatz analysis. Thus, let us turn to the problem of the diagonalization of $H_2$. First of all, it is possible to show that this model is obtained as the logarithmic derivative of the following transfer matrix:

$$T(\lambda) = Tr_0[\mathcal{L}_{0L}(\lambda) \ldots \mathcal{L}_{01}(\lambda)] \tag{9}$$

where the index 0 denotes the $4 \times 4$ auxiliary space and the operator $\mathcal{L}(\lambda)_{ab}^{cd} = (-1)^{p(a)p(b)} R(\lambda)_{ba}^{cd}$. If we choose the particular $fbbf$ grading, we find that following expression for $\mathcal{L}(\lambda)$:

$$\mathcal{L}(\lambda) = \begin{pmatrix} \begin{matrix} l(\lambda)e_{11} + p(\lambda)e_{44} \\ +\lambda[e_{22} + e_{33}] \end{matrix} & e_{21} - \sigma(\lambda)e_{42} & e_{31} + \sigma(\lambda)e_{42} & n(\lambda)e_{41} \\ e_{12} - \sigma(\lambda)e_{34} & \begin{matrix} \lambda[e_{11} + e_{44}] \\ +a(\lambda)e_{22} + t(\lambda)e_{33} \end{matrix} & s(\lambda)e_{32} & e_{42} - \sigma(\lambda)e_{31} \\ e_{13} + \sigma(\lambda)e_{24} & s(\lambda)e_{23} & \begin{matrix} \lambda[e_{11} + e_{44}] \\ +t(\lambda)e_{22} + a(\lambda)e_{33} \end{matrix} & e_{43} - \sigma(\lambda)e_{21} \\ n(\lambda)e_{14} & e_{24} - \sigma(\lambda)e_{13} & e_{34} + \sigma(\lambda)e_{12} & \begin{matrix} p(\lambda)e_{11} + l(\lambda)e_{44} \\ +\lambda[e_{22} + e_{33}] \end{matrix} \end{pmatrix} \tag{10}$$

where the matrix elements of $(e_{ab})_{cd} = \delta_{ac}\delta_{bd}$ and the functions appearing in (10) are given by

$$\begin{aligned} &l(\lambda) = 1 - \lambda \qquad n(\lambda) = \frac{1}{1+\lambda} \qquad \sigma(\lambda) = \lambda n(\lambda) \qquad a(\lambda) = \frac{1}{n(\lambda)} \\ &s(\lambda) = (1 + 2\lambda)n(\lambda) \qquad p(\lambda) = -2\lambda\left(1 + \frac{\lambda}{2}\right)n(\lambda) \qquad t(\lambda) = \lambda^2 n(\lambda) \,. \end{aligned} \tag{11}$$

In order to apply the analytical Bethe ansatz we first notice that the action of the operator $\mathcal{L}(\lambda)$ in the usual ferromagnetic reference state has a triangular form, namely,

$$\mathcal{L}(\lambda)|0\rangle = \begin{pmatrix} 1-\lambda & * & * & * \\ 0 & \lambda & 0 & * \\ 0 & 0 & \lambda & * \\ 0 & 0 & 0 & -\frac{\lambda(2+\lambda)}{1+\lambda} \end{pmatrix} \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \tag{12}$$

Taking into account this structure, the analytical Bethe ansatz approach (see e.g. [18]) now seeks for a more general ansatz for the eigenvalues of $T(\lambda)$ as

$$\begin{aligned} \Lambda(\lambda; \lambda_j^1, \lambda_j^2) = (1 - \lambda)^L \prod_{j=1}^{M_1} A(\lambda - \lambda_j^1) + \lambda^L \sum_{k=1}^{2} \prod_{j=1}^{M_k} B(\lambda, \lambda_j^k, \lambda_j^{k+1}) \\ + \left[ -\frac{\lambda(2+\lambda)}{\lambda+1} \right]^L \prod_{j=1}^{M_1} C(\lambda - \lambda_j^1) \end{aligned} \tag{13}$$

where the rational functions $A(x)$, $B_j(x)$ and $C(x)$ can be fixed by exploiting crossing symmetry†, unitarity condition and the asymptotic behaviour of the operator $\mathcal{L}(\lambda)$. We then have found the following relations:

$$A(-1-x) = C(x) \qquad B_1(-1-x) = B_2(x) \qquad A(x)A(-x) = 1\,. \tag{14}$$

Considering the 'minimal' pole assumption [18] (and our previous experience with $Osp(1|2n)$ systems [20]) we start with the ansatz $A(x) = \frac{x+1/2}{x-1/2}$. Now, by using properties (12) and by looking for a common pole structure in the $fbbf$ grading for $\Lambda(\lambda; \lambda_j^1, \lambda_j^2)$, we are able to find the following solution:

$$\begin{aligned}\Lambda(\lambda; \lambda_j^1, \lambda_j^2) = {} & (\mathrm{i}-\lambda)^L \prod_{j=1}^{M_1} \frac{\lambda - \lambda_j^1 + \mathrm{i}/2}{\lambda - \lambda_j^1 - \mathrm{i}/2} + \lambda^L \left\{ \prod_{j=1}^{M_1} \frac{\lambda - \lambda_j^1 + \mathrm{i}/2}{\lambda - \lambda_j^1 - \mathrm{i}/2} \right. \\ & \left. \prod_{j=1}^{M_2} \frac{\lambda - \lambda_j^2 - \mathrm{i}3/2}{\lambda - \lambda_j^2 + \mathrm{i}/2} + \prod_{j=1}^{M_1} \frac{\lambda - \lambda_j^1 + \mathrm{i}/2}{\lambda - \lambda_j^1 + \mathrm{i}3/2} \prod_{j=1}^{M_2} \frac{\lambda - \lambda_j^2 + \mathrm{i}5/2}{\lambda - \lambda_j^2 + \mathrm{i}/2} \right\} \\ & + \left[ -\frac{\lambda(2\mathrm{i}+\lambda)}{\lambda + \mathrm{i}} \right]^L \prod_{j=1}^{M_1} \frac{\lambda - \lambda_j^1 + \mathrm{i}/2}{\lambda - \lambda_j^1 + 3\mathrm{i}/2}\end{aligned} \tag{15}$$

where for later convenience the scaling $\lambda \to \mathrm{i}\lambda$ has been used. Finally, the set of numbers $\{\lambda_j^1, \lambda_j^2\}$ are fixed by imposing that the eigenvalues $\Lambda(\lambda, \lambda_j^1, \lambda_j^2)$ have no pole at finite value of $\lambda$. This leads to the following Bethe ansatz condition:

$$\left( \frac{\lambda_j^1 - \mathrm{i}/2}{\lambda_j^1 + \mathrm{i}/2} \right)^L = -(-1)^L \prod_{k=1}^{M_2} \frac{\lambda_j^1 - \lambda_k^2 - \mathrm{i}}{\lambda_j^1 - \lambda_k^2 + \mathrm{i}} \qquad - \prod_{k=1}^{M_2} \frac{\lambda_j^2 - \lambda_k^2 - 2\mathrm{i}}{\lambda_j^2 - \lambda_k^2 + 2\mathrm{i}} = \prod_{k=1}^{M_1} \frac{\lambda_j^2 - \lambda_k^1 - \mathrm{i}}{\lambda_j^2 - \lambda_k^1 + \mathrm{i}} \tag{16}$$

and the eigenenergies of Hamiltonian (8) are parametrized by

$$E(L) = \sum_{j=1}^{M_1} \frac{1}{(\lambda_j^1)^2 + \frac{1}{4}} - L\,. \tag{17}$$

Such results are almost in accordance to that obtained previously in [3], if one considers the special rational case and the limit $b \to 0$ (a fundamental four-dimensional representation of $slp(2|1)$) in Maassarini's results. In particular, our analytical result for the eigenvalue $\Lambda(\lambda, \lambda_j^1, \lambda_j^2)$ gives further support to the conjecture made in [3] concerning its general structure for the $spl(2|1)$ model. The basic difference is the presence of an extra factor $-(-1)^L$ in our equations (13) and (16). We believe that its origin is a consequence of the fact that we have not used the supertensor formalism of [3]. We find that this factor is crucial in order to fit the ground-state property of the Hamiltonian (8). We have verified this fact (even for odd lattices) by numerically solving the Bethe ansatz equations (16), (17) and have compared them with the exact diagonalization of Hamiltonian (8) for several values of $L$. Remarkably enough, this analysis leads us to find that the roots $\{\lambda_j^1, \lambda_j^2\}$ governing the ground state have a special string behaviour. This is shown in table 1 for some intermediate zeros and several values of $L$. Our numerical results suggest the following structure of solution for the ground state:

$$\lambda_j^1 = \xi_j \pm \mathrm{i} + \mathrm{O}(\mathrm{e}^{-aL}) \qquad \lambda_j^2 = \xi_j \tag{18}$$

where $a > 0$ and $j = 1, \ldots, [L/2]$‡. In the thermodynamic limit, $L \to \infty$, the roots $\{\lambda_j^1\}$ are believed to cluster around $\pm\mathrm{i}$. In this case the second Bethe ansatz equation is automatically

† In our case the crossing property acts as $R_i(\lambda) = (\alpha_2 \otimes I) R_i^{st_i}(-1-\lambda)(\alpha_2 \otimes I)^{st_i}$.

‡ $[L/2]$ is the integer part of $L/2$.

**Table 1.** Some intermediate (middle of the chain) zeros corresponding to the ground-state structure of (16).

| L | $\mathrm{Im}[\lambda^1]$ | $\mathrm{Re}[\lambda^1]$ | $\lambda^2$ |
|---|---|---|---|
| 16 | 0.971 66 | 0.280 74 | 0.282 36 |
| 20 | 0.973 07 | 0.379 49 | 0.381 79 |
| 24 | 0.983 95 | 0.280 75 | 0.281 40 |
| 28 | 0.983 95 | 0.344 14 | 0.344 84 |
| 32 | 0.988 56 | 0.327 31 | 0.327 70 |

satisfied. This situation resembles those appearing in the supersymmetric $T-J$ model [21], and some care has to be taken to perform the limit in the first equation (16). We recall, however, that our case is a bit more involved since the roots cluster around $\pm\mathrm{i}$. Considering previous experience with the $t-J$ model [21] and our string structure (18) we find the following final form for the Bethe ansatz (governing the ground state):

$$-(-1)^L \mathrm{e}^{\mathrm{i}L\psi(\xi_j)} = \prod_{j\neq k}^{L/2} \mathrm{e}^{\mathrm{i}\phi(\xi_j-\xi_k)} \tag{19}$$

where $\psi(x) = 2[\arctan(2x/3) - \arctan(2x)]$ and $\phi(x) = 2\arctan(x/2)$. Thus, the final effect of the ansatz (18) is that we end up with a scattering between particles with pseudo-momentum $\psi(\xi)$ and phase-shift $\mathrm{e}^{\mathrm{i}\phi(\xi)}$ around a ring of size $L$. Strictly in the thermodynamic limit, equation (19) goes to a integral equation for the density $\rho(\xi)$

$$\psi'(\xi) + 2\pi\rho(\xi) = \int_{-\infty}^{+\infty} \phi'(\xi-u)\rho(u)\,\mathrm{d}u \tag{20}$$

where the prime symbol stands for the derivative. Such an integral equation is then solved by elementary Fourier techniques, and we find the following simple result:

$$\rho(\xi) = \frac{1}{\cosh(\pi\xi)} \tag{21}$$

and the ground-state energy per particle $e_\infty$ is given by

$$e_\infty = \int_{-\infty}^{\infty} \rho(\xi)\psi'(\xi)\,\mathrm{d}\xi - 1 = -4\ln(2) + 1\,. \tag{22}$$

In order to conclude this paper we would like to remark on the possible interpretation of Hamiltonian (8) in terms of strongly correlated electronic systems [4, 11–14]. Recently, new generalizations of the Hubbard model containing additional hopping terms have been proposed in the context of the rational $gl(2|1)$ vertex model [4] and a generalization form of six-vertex scattering [12–14]. We have verified that the two-body term of our second Hamiltonian ($H_2$) corresponds to an analytical continuation in the non-Hermitian region of this general Hubbard model [4, 13]. In particular, in the notation of [4], our two-body term of $H_2$ is seen as the limit $U \to -2$, where $U$ is Hubbard parameter entering in [4]. The first Hamiltonian, however, presents some new terms. More precisely, by taking the basis of the four electrons states as follows:

$$|0\rangle \qquad c_{i,+}^{\dagger}|0\rangle \qquad c_{i,-}^{\dagger}|0\rangle \qquad c_{i,+}^{\dagger}c_{i,-}^{\dagger}|0\rangle \tag{23}$$

we find that $H_1$ contains the following two extra terms:

$$\sum_{\sigma=\pm} \sigma[c_{i,-\sigma}^{\dagger}c_{i+1,\sigma} + c_{i,-\sigma}c_{i+1,\sigma}^{\dagger}][n_{i,\sigma} + n_{i+1,\sigma} - 2n_{i,\sigma}n_{i+1,-\sigma}] \tag{24}$$

and

$$[n_{i,+} - n_{i,-}][n_{i+1,+} - n_{i+1,-}] \tag{25}$$

where $n_i, +(n_i, -)$ is the number of electrons up (down). Interesting enough, we notice the presence of an off-site Coulomb interaction. The price we have paid is the explicit breaking (equation (24)) of the $U(1)$ symmetry in order to guarantee integrability. Although these two Hamiltonians share the same eigenvalues, they do not necessarily have the same eigenfunction. In fact, a numerical analysis reveals that even the ground-state wavefunction is, in fact, different in these two models. This means that such systems may present different behaviour concerning correlation functions and therefore are related to distinct physical behaviour. Concerning physical applications, the main drawback of these models is that they are intrinsically non-Hermitian.

In summary we have discussed equivalent $Osp(2|2)$ spin chains by the braid-monoid approach. We have used the analytical Bethe ansatz approach in order to determine the ground-state energy. Motived by these results, additionally we have been able to generalize the $U(1)$ monoid construction for all $Osp(n|2m)$ algebra. In this case the $U(1)$ monoid is built on an antidiagonal matrix $\alpha$ possessing only elements $\pm 1$. The integer $m$ is the number of minus signs and $n$ = antitrace($\alpha$). The first non-trivial case of such generalization is in fact the $Osp(2|2)$ system, since the $Osp(1|2n)$ is automatically $U(1)$ invariant [20]. Hopefully, this new construction will be useful in order to find the Bethe ansatz solution for the general $Osp(n|2m)$ $R$-matrices.

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